Some Exercises on Pareto Aggregation

Assume throughout that P is an irreflexive relation on the set X; that $I := P^c \cup (P^{-1})^c$ – i.e., that xIy if and only if neither xPy nor yPx; and that $R := P \cup I$ – i.e., that xRy if and only if xPy or xIy. (P^c denotes the complement of P – i.e., xP^cy if and only if "not xPy.") For any list $(P_1, ..., P_n)$ of preference relations, let $\overline{P}$ denote the associated Pareto relation, i.e., the Pareto aggregation of $(P_1, ..., P_n)$.

Exercise 1: Prove that the relation R is transitive if and only if its associated P and I are both transitive.

Exercise 2: Prove that if, for each $i \in N$, P_i is irreflexive, then $\overline{P}$ is irreflexive.

Exercise 3: Provide a counterexample to the following proposition: "If, for each $i \in N$, P_i is transitive, then $\overline{P}$ is transitive." (Try to find the simplest possible counterexample. It might help to use the interpretation that the elements of X are universities, or economics departments, or basketball teams, etc. It may also help in this case to remember that a binary relation on a set X is a subset of $X \times X$.)

Exercise 4: Prove that if, for an irreflexive relation P, the associated R is transitive, then

(i) xPy & $yRz \Rightarrow xPz$

(ii) xRy & $yPz \Rightarrow xPz$.

Exercise 5: Prove that if, for each $i \in N$, R_i is transitive, then $\overline{P}$ is transitive.

The lecture notes provide examples which show that if each R_i is transitive, $\overline{I}$ need not be transitive, and thus, according to Exercises 1 and 2, $\overline{R}$ need not be transitive.